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Initial conditions for inflation

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Abstract: Free scalar fields in de Sitter space have a one-parameter family of states invariant under the de Sitter group, including the standard thermal vacuum. We show that, except for the thermal vacuum, these states are unphysical when gravitational interactions are included. We apply these observations to the quantum state of the inflaton, and find that at best, dramatic fine tuning is required for states other than the thermal vacuum to lead to observable features in the CMBR anisotropy.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe.

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Contents

1. Introduction

In inflationary cosmology, cosmic microwave background (CMB) data place a tantalizing upper bound on the vacuum energy density during the inflationary epoch:

$$V \sim M_{\mathrm{GUT}}^4 \sim \left(10^{16}\,\mathrm{GeV}\right)^4 . \tag{1.1}$$

Here M_{GUT} is the "unification scale" in supersymmetric grand unified models, as predicted by the running of the observed strong, weak and electromagnetic couplings above 1 TeV in the minimal supersymmetric standard model. If this upper bound is close to the truth, the vacuum energy can be measured directly with detectors sensitive to the polarization of the CMBR.

In these scenarios, CMBR anisotropies are generated by quantum fluctuations of the inflaton and graviton which "freeze" into classical perturbations at the inflationary Hubble scale, $H \sim 10^{14}$ GeV. These fluctuations are inflated to observable scales over the 65 e-foldings required to generate the observed homogeneity and isotropy of the CMBR. Therefore, inflation acts as both accelerator and microscope, potentially to energies 11 orders of magnitude higher than those detectable by terrestrial accelerators.

To this end, a number of authors [1]–[5] and [4, 5] have argued that high-scale physics can lead to observable effects in the CMBR. Following this, four of the present authors [6] undertook a systematic and general study of the effects on CMB observations of new physics at a scale $M \geq H$. Our conclusions differed from some of the previous work. Assuming that local, effective field theory applies at the scale H, we argued that short-distance effects were encapsulated in irrelevant operators in the inflaton lagrangian. Higher-derivative terms lead to effects which are distinguishable from corrections to the effective potential, by modifying a relationship (one of the inflationary "consistency conditions" [7]) between the temperature and polarization anisotropies of the CMB.[1] Locality and Lorentz invariance imply that the effects are of order $(H/M)^2$.

Unlike some of the previous work, the calculations in [6] assumed that the quantum state of the inflaton and graviton at scales close to H was the standard thermal[2] vacuum of de Sitter space.[3] The result is that our effects are smaller than those discussed in the work cited above; in particular, in [1, 4, 5], the effects are of order H/M(the results of [3], however, agree with [6]). For this reason, the results of [6] have been criticized as being "too pessimistic" [13]–[16]. [4]

Generic excitations above the thermal vacuum tend to inflate away — indeed, this is part of the virtue of inflation. But as pointed out in [16], the quantum states advocated by [1, 4, 5, 13, 14] as an initial state for inflation lie in a one-parameter family of de Sitter-invariant states, constructed and described in [17]–[20]. (See [21] for a particularly clear discussion of these states). These states, as we will describe below in detail, resemble squeezed states or Bogoliubov rotations of the standard, thermal vacuum. Relative to this vacuum, they have excitations at arbitrarily high energies. In pure de Sitter space, they make up a family of de Sitter-invariant states $|\alpha\rangle$, where α is a complex parameter indexing the state; (thus they are often called the "α-vacua"). These states appear to result in an α-dependent correction to the standard formulae for CMB anisotropies. The effect is of order H/M, where M is the scale of new physics, which is larger than the H^2/M^2 effects discussed in [6, 3].

[1]This is not the only way to violate these consistency conditions; scenarios with multiple scalar fields also lead to a violation [8]–[12]. However, as argued in [6], these deformations require drastic fine tuning of both the potential energy functional and the initial conditions for these fields.

[2]The standard de Sitter vacuum is referred to variously as the thermal, adiabatic, and Bunch-Davies vacuum; we will use thermal.

[3]Of course the inflationary universe is not quite de Sitter space, as the Hubble constant is changing, but so long as the "slow roll" approximation applies, dynamics at length scales on the order of or above the several times the Hubble scale can be approximated as occurring in de Sitter space.

[4][13, 14] also find that the correction due to the α state will be sinusoidal in the Hubble parameter, and so in observed wavenumber at the end of inflation. This would be a striking signature.

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The states $|\alpha\rangle$ have also made an appearance in work on the conjectured correspondence between euclidean conformal field theory and quantum gravity in de Sitter space [21]. Changing α is conjectured to correspond to a marginal deformation of the dual CFT. This makes them doubly interesting.

Motivated primarily by the possible utility of these states as inflationary initial conditions, we would like to examine them more closely. We will find that these states, while perhaps consistent with an absolutely free field in a fixed background de Sitter space, are inconsistent in inflationary cosmology, and indeed are problematic even in exact de Sitter space once gravitational backreaction is included.

With some optimism about the future sensitivity of CMB experiments, especially polarization data, [6] estimated that the deviations from standard predictions would be observable if the correction — be it (H/M) or $(H/M)^2$ — is of order 10^{-1}. Since the difference between scales M detected in each case is then a factor of a few, this difference may seem academic. But future gravitational wave detectors may be more sensitive, because they are not limited by cosmic variance constraints. However, uncertainties in the backgrounds make their ultimate sensitivity hard to predict (see [22]–[24]). If these measurements were sensitive at the 10^{-4} level, the difference in scales probed by H/M effects versus H^2/M^2 would be quite substantial. So it is experimentally, as well as theoretically, important to resolve this issue.

1.1 Summary

Before continuing, we would like to provide a summary of our basic arguments. The usual attitude amongst inflationary theorists is that any deviation from the thermal vacuum relaxes to the thermal vacuum within a few e-foldings of inflation. But in the states $|\alpha\rangle$, the deviations from the thermal vacuum do not inflate away, because these deviations extend to arbitrarily short distances. As a direct consequence, the quantum energy-momentum tensor $\langle\alpha|T_{\mu\nu}|\alpha\rangle$ diverges.

Of course, the computation of $\langle T_{\mu\nu}\rangle$ diverges even in the thermal vacuum, for the same reason, and with the same coefficient, as in Minkowski space. A consistent regulator in the thermal vacuum is therefore also a consistent regulator in flat space; the short-distance properties of Green functions are the same. However, $\langle\alpha|T_{\mu\nu}|\alpha\rangle$ diverges with a coefficient that is α-dependent, as the short-distance behavior of Green functions computed with $|\alpha\rangle$ differs from those in flat space.

We can choose an (α-dependent) regulator, such that $\langle\alpha|T_{\mu\nu}|\alpha\rangle$ is finite, but then the energy-momentum tensor will diverge in Minkowski space, and in the thermal vacuum of de Sitter. Of course, the universe today may be approximately de Sitter, and *a priori* could therefore still be in an α state. However, we will show that values of α which cause interesting effects on the CMB spectrum produced during inflation would produce enormously unphysical effects on physics in the world today.

A clear sign that the states $|\alpha\rangle$ are fundamentally sick is the response of an inertial detector coupled linearly to a field in the α state [25, 21]. The detector will equilibrate to a non-thermal spectrum containing infinite energy excitations, which are unsuppressed

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by any Boltzman factor. This is incompatible with the "causal patch" description of de Sitter space. In such coordinates, physics close to the horizon is best described by a temperature $T = M_p$ "stretched horizon", where gravitational couplings are order one [26]. The stretched horizon acts as a universal thermalizer. The system is a finite energy, closed, thermal cavity, which will always equilibrate and thermalize its contents, with the thermalization time scale set in this case by the time it takes an excitation to fall away from the center of the patch and reach the stretched horizon. The non-thermal α state can either equilibrate with the walls (resulting in a thermal vacuum), or is simply inconsistent due to back-reaction from the infinite energy. The same class of arguments also show that black hole radiation must be thermal, despite the apparent appearance of transplanckian modes near the horizon. The thermal nature of black hole radiation is well-tested in string theory, and is particularly manifest in studies of the AdS black hole.

This leaves the thermal vacuum, and finite energy excitations above it, as the only consistent choice for the initial quantum state of an interacting field theory in an inflating universe. Since excitations inflate away and thermalize, the vacuum is the most generic choice. Nonetheless, rather than considering the exact α state, one can consider an excited state above the thermal vacuum, for example one which is identical to the α state up to some high scale E, and is then populated thermally at higher scales. This state will have a very large energy if E is large, and hence will back-react and invalidate the model. Even if this difficulty is ignored, or the scale is low enough that it is not a problem, the energy E will exponentially decrease with time, leading rapidly to a near exact thermal state. Therefore, in order to produce interesting effects on the CMB spectrum, this initial state would have to be fine-tuned to be important for precisely the 10 e-foldings visible in the fluctuation spectrum.

The last possibility is to simply cut off the quantum contributions to $\langle T_{\mu\nu}\rangle_\alpha$ above a fixed, physical scale M. This is the attitude taken in [4], where M is taken to be the mass scale where new, unknown physics becomes important. Of course, any excitations below the scale M will inflate away within order $\ln M/H$ e-foldings, in the same manner and for the same reason more conventional perturbations of the inflationary initial conditions do. Therefore, in order to produce effects that last longer than a few initial e-foldings, one has to postulate that the new physics at scale M "pumps" in new modes in an α configuration for all times during inflation. As these modes are produced at M and inflate away, more must be created to prevent the effect from disappearing. It is possible to choose M and α such that the extra energy density in the modes below the scale M is small compared to the curvature during inflation. However, this "pump" must shut itself off at the end of inflation, as otherwise the extra energy would dominate the post-inflation universe. This appears to be both fine-tuned and a violation of standard wilsonian decoupling. Furthermore, the physics above M can not be simply ignored — in particular, conservation of energy implies that the modes must be produced out of some energy reservoir, which should gravitate strongly in much the same way as the original α state did. Finally, our understanding of string- and M-theory, incomplete as it may be, provides no hint of any such behavior; on the contrary, the stretched horizon (which provides a cut-off much like this) is explicitly thermal.

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1.2 Outline

The remainder of this paper is as follows. In section 2 we will discuss the states $|\alpha\rangle$ in eternal de Sitter space. In the beginning of section 2 we will discuss these states in FRW-type coordinates, and argue that any consistent regularization will cause the differences between $|\alpha\rangle$ and the thermal vacuum to inflate away. Section 2.2 will discuss the complementary picture of this argument in the static patch, and argue that the state $|\alpha\rangle$ corresponds to a non-thermal state which rapidly thermalizes. In section 3 we will apply these observations to an inflationary universe. We then conclude, and discuss some details of the thermal character of the α states in the appendix.

1.3 Notes on previous work

The fact that the α states are totally unsuited for use as initial conditions for inflation has been noted at least as far back as [17], in which the author mentions that the thermal vacuum is the correct choice for a realistic inflation model, for essentially the same reasons we are explaining here. Indeed, many of the arguments we present here, including the analogy between inflationary fluctuations and Hawking radiation, informed the general consensus within the inflation community that the thermal vacuum is the correct one.[5]

After this work was completed, we received [27] and [28] which argue that interacting field theory in the α state is not well defined, due to the appearance of non-local divergences at one loop. In addition, [28] reached conclusions identical to ours on the thermal character of the α states. In another related work [29], the authors concluded that the states $|\alpha\rangle$ lead to infinitely large backreaction. The authors then go on to consider a non-de Sitter-invariant state which leads to corrections to the CMB spectrum at order H/M. We discuss such states in §2, which arise when one throws away modes of $|\alpha\rangle$ above some frequency. As we point out there, states which do not lead to unacceptable backreaction inflate away within a few e-foldings. In order to produce a sufficiently flat and homogeneous universe, inflation needs to have lasted for at least 65 e-foldings. Therefore, the state considered in [29] would involve energies at least up to $e^{65}M$, where M is some high scale. Such states, if regulated in a way consistent with either the thermal vacuum or any α state, would produce an enormous backreaction which would invalidate the model. Even if a non-de Sitter invariant regulator could be found that would remove this divergence, in sec. 3 we point out that a fine tuning is required for these states to affect CMBR measurements. Unless the UV scale considered in [29] is tuned precisely to a time corresponding to the observable part of inflation, there will be no effect on the spectrum.

2. The states $|\alpha\rangle$ in de Sitter space

In this section we will argue that the states $|\alpha\rangle$ are not consistent in a field theory interacting with gravity in de Sitter space. It is useful to study this issue from several points of view.

The principle of equivalence states that physics is invariant under the choice of coordinate system. Nonetheless, the description may be very different. We have become

[5]We would like to thank A. Albrecht and S.-H. Tye for discussions on this point.

used to this fact in studies of black holes. From the standpoint of an observer at fixed distance outside a black hole, an infalling object is blueshifted as it approaches the horizon and eventually reaches planckian energies. A “stretched horizon” [26] can be defined as the place at which modes which asymptotically have energies equal to the Hawking temperature are blueshifted to the Planck scale. This stretched horizon can be treated as a thermal membrane, and an infalling observer reaching it will rapidly thermalize via gravitational and other interactions. On the other hand, infalling obsevers will fall through the horizon and not need planckian physics to describe this experience. That these two pictures are equivalent is demanded by general covariance, and is the statement of black hole complementarity [26].

Similar points of view exist in de Sitter space. Note first that the causal structure — the Penrose diagram — is the same as that of the AdS-Schwarzchild black hole, a fact noted by many physicists. There are static coordinates which cover the single “causal diamond” that an inertial observer can access in experiments; these are analogous to Schwarzchild coordinates in a black hole background. Physics near the horizon can be described by a thermal “stretched horizon” just as it is for the black hole. There are also coordinates describing all parts of de Sitter space which can be influenced by an inertial observer; these are the analogue of coordinates which can describe an infalling observer. The two pictures are complementary. The statement in flat coordinates that an excitation inflates away is related to the statement in static coordinates that excitations rapidly thermalize when they approach the causal horizon. The precise thermality of fluctuations in de Sitter space follows for the same reason that the Hawking radiation of a black hole is believed to be thermal.

In the remainder of this section we will analyze the states $|\alpha\rangle$ from both points of view.

2.1 $|\alpha\rangle$ in flat coordinates

Inflation is conventionally described in flat FRW coordinates (c.f. [30]–[33]). The Robertson-Walker form of the metric is:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \,. \tag{2.1}$$

Here $\vec{x}$ is a 3-vector, and $(t, \vec{x})$ take values in all of $\mathbb{R}^4$. H is the “Hubble scale”, and objects separated by a comoving (x-coordinate) spatial distance larger than e^{-t}/H are out of causal contact with each other. These coordinates cover the half of de Sitter space labelled I in the figure 1.

Using the transformation

$$\eta = -\frac{1}{H} e^{-Ht} \,, \tag{2.2}$$

this same patch can be described in conformally flat coordinates using the metric

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right) , \tag{2.3}$$

where $\eta \in (-\infty, 0)$.

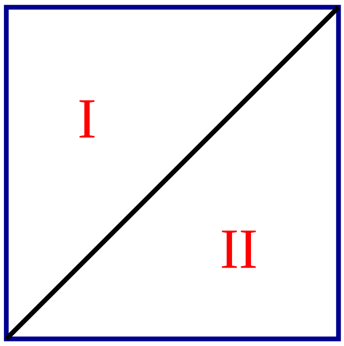

Figure 1: Region I is the spatially flat patch of de Sitter space for an observer on the left-hand boundary of the diagram.

2.1.1 Constructing the states

Let ϕ be a free, minimally coupled scalar field with mass m. This could for example be the inflaton, for which $m \ll H$. A complete set of solutions to the Klein-Gordon equation in conformal coordinates is [34]:

$$\phi(\eta, \vec{x})_k^{\substack{+\\-}} = \frac{H\pi^{1/2}\eta^{3/2}}{2(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} H_\nu^{\binom{2}{1}}(k\eta)\,, \tag{2.4}$$

where

$$\nu = \sqrt{\frac{9}{4} - 12\frac{m^2}{H^2}}\,, \tag{2.5}$$

$k = |\vec{k}|$, and $H_\nu^{\binom{1}{2}}$ are Hankel functions of the first and second kind. As $k \to \infty$ (or $\eta \to -\infty$),

$$H^{\binom{2}{1}}(k\eta) \to \left(\frac{2}{\pi k\eta}\right) e^{\binom{-}{+}ik\eta}\,, \tag{2.6}$$

which is the standard behavior for a positive (negative) frequency mode in flat space. As expected, the curvature corrections disappear at large momentum.

The thermal vacuum corresponds to choosing $H^{(2)}$ as the positive frequency modes. In other words, one may decompose the field operator ϕ as

$$\phi = \sum_k \left(\phi_k^+ a_k + \phi_k^- a_k^\dagger\right)\,, \tag{2.7}$$

where $a_k, a_k^\dagger$ are creation and annihilation operators satisfying the usual canonical commutation relations. The standard thermal vacuum then satisfies

$$a_k|\text{thermal}\rangle = 0\,. \tag{2.8}$$

The motivation for choosing this vacuum is that at short distances it looks like the vacuum in flat space, in accord with the standard intuition that the short-distance behavior of the theory should be independent of the space-time curvature.

The thermal vacuum lives in a one-complex parameter family of states which are invariant under the isometry group of de Sitter space [17]–[20] (see [21] for a recent, clear discussion). This family can be described via Bogoliubov transformations as follows [17, 18, 21]. For complex parameter α, $\mathrm{Re}(\alpha) < 0$, the new annihilation operators are [21]:

$$a_k^\alpha = N_\alpha \left(a_k - e^{\alpha^*} a_k^\dagger \right) , \tag{2.9}$$

where

$$N_\alpha = \frac{1}{\sqrt{1 - \exp(\alpha + \alpha^*)}} , \tag{2.10}$$

and

$$a_k^\alpha |\alpha\rangle = 0 .$$

The thermal vacuum is the limit $\mathrm{Re}(\alpha) \to -\infty$ of these states. Such a choice corresponds, in the high momentum limit (2.6), to choosing a mixed set of positive and negative frequency modes for the creation and annihilation operators. It is this fact that underlies all of the difficulties the states $|\alpha\rangle$ suffer from.

2.1.2 Two-point functions

One may express the various Green functions in terms of the unordered or Wightman two-point functions (c.f. [34].) The Wightman functions in the state $|\alpha\rangle$

$$G_\alpha(x, x') = \langle \alpha | \phi(x) \phi(x') | \alpha \rangle \tag{2.11}$$

have a simple expression in terms of the thermal Wightman function

$$G_T = \langle 0 | \phi(x) \phi(x') | 0 \rangle . \tag{2.12}$$

To construct G_α, we must continue G_T to global de Sitter space, defined as the the hyperboloid

$$-(X^0)^2 + \sum_{k=1}^{d} (X^k)^2 = \frac{1}{H^2} \tag{2.13}$$

in $(d+1)$-dimensional Minkowski space $\mathbb{R}^{1,d}$. Define the antipodal point of $X(x)$ as $X_A = -X$. Then G_α can be written as [21]:

$$\begin{aligned} G_\alpha(x, x') = |N_\alpha|^2 \Big[& G_T(x, x') + \exp(\alpha + \alpha^*)\, G_T(x', x) + \\ & + \exp(\alpha^*) G_T(x, x'_A) + \exp(\alpha) G_T(x_A, x') \Big] . \end{aligned} \tag{2.14}$$

The short-distance behavior of G_α is different for each α. For distances much smaller than the Hubble scale, G_T looks like the unordered two-point function in the Minkowski space vacuum. At these distances one can use Minkowski coordinates $t, \vec{x}$, and the Wightman function approaches:

$$G_T \sim \frac{1}{[(t - t' - i\epsilon)^2 - |\vec{x} - \vec{x}'|^2]^{d/2-1}} . \tag{2.15}$$

From this it is easy to see that at short distances:

$$\begin{aligned}\text{Re } G_\alpha &\sim \frac{1+\exp(\alpha+\alpha^*)}{1-\exp(\alpha+\alpha^*)}\text{Re } G_T \\ \text{Im } G_\alpha &\sim \text{Im } G_T\,.\end{aligned} \tag{2.16}$$

The Wightman functions can be used to construct the commutator and anticommutator of ϕ and the usual Green functions (c.f. [34].) The commutator

$$\langle\alpha|\left[\phi(t,x),\partial_t\phi(t,x')\right]|\alpha\rangle = \lim_{t\to t'}\partial_{t'}\left(G_\alpha(x,x')-G_\alpha(x',x)\right) \tag{2.17}$$

can be shown using (2.14) to be independent of α, as expected since the free field commutator is a c-number and is determined uniquely by the operator algebra. The Feynman propagator

$$\begin{aligned}G_{F,\alpha} &= -i\theta(t-t')G_\alpha(x,x')-i\theta(t'-t)G_\alpha(x',x)\\ &= \frac{1}{2}\left[\theta(t-t')G_\alpha(x,x')-\theta(t'-t)G_\alpha(x',x)\right]-\frac{i}{2}\left[G_\alpha(x,x')+G_\alpha(x',x)\right]\end{aligned} \tag{2.18}$$

has different short-distance behavior from $G_{F,0}$, however. The first term in square brackets is determined by the operator algebra and is therefore independent of the state, but the second term — the Hadamard function, which is determined by the anti-commutator of the fields — is α dependent.

2.1.3 Particle content of $|\alpha\rangle$.

We can measure the structure of $|\alpha\rangle$ by coupling ϕ to an inertial detector. Since no stress-energy is needed to maintain the trajectory of an inertial observer, any transition the detector makes must be the result of absorbing quanta of the field ϕ.

The "Unruh detector" [25] couples linearly to ϕ,

$$\delta H = \int d\tau\sqrt{g}m(\tau)\phi(t(\tau),\vec{x}(\tau))\,, \tag{2.19}$$

where $m(\tau)$ is some operator measuring the state of a detector, and $t(\tau),\vec{x}(\tau)$ is the trajectory. Let us imagine a detector at the origin in inflating coordinates, so that the coordinate time and the proper time along the detector's trajectory are identical. In this case, the response of an Unruh detector was calculated for all values of α in [21].[6] If the initial state of ϕ is $|\alpha\rangle$,

$$\dot{P}_{\alpha,i\to j} = \langle E_j|m(0)|E_i\rangle\int_{-\infty}^{\infty}dte^{-i\delta Et}G_\alpha(t) \tag{2.20}$$

is the probability per unit time for the detector to make a transition from a state with energy E_i to a state with energy $E_j = E_i+\delta E$. We can use general properties of G_T under translations in imaginary time [21] to compute the ratio for $\delta E \gg H/(2\pi)$:

$$\mathcal{R}_{ij} = \frac{\dot{P}_{i\to j}}{\dot{P}_{j\to i}} = e^{-2\pi\delta E/H}\left|\frac{1+e^{\alpha+\pi\delta E/H}}{1+e^{\alpha-\pi\delta E/H}}\right|^2 = f_\alpha(\delta E)\,. \tag{2.21}$$

This ratio is finite even if we do not regularize G_α.

[6] The authors of [21] computed this result for 2+1 dimensional de Sitter space. However, the computation can be easily extended to 3+1 dimensions, with the same result for the spectrum of the Unruh detector.

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The thermal vacuum is the limit $\alpha \to -\infty$. In this state, the detector will equilibrate to

$$\frac{\dot{P}_{i\to j}}{\dot{P}_{j\to i}} = \frac{\rho(E_j)}{\rho(E_i)} = e^{-2\pi\delta E/H}\,, \tag{2.22}$$

where $\rho(E)$ is the density of states at energy E. The detector is in equilibrium at temperature $T_{dS} = H/(2\pi)$, which is consistent with the usual statement that the standard vacuum is thermal, with temperature T_{dS}.

If $\alpha \neq -\infty$, however, the detector cannot reach an equilibrium consistent with the principle of detailed balance. Detailed balance combined with (2.21) implies:

$$\mathcal{R}_{ij} = \frac{\rho(E_j)}{\rho(E_i)}\,. \tag{2.23}$$

But this implies

$$\mathcal{R}_{ij}\mathcal{R}_{jk} = \mathcal{R}_{ik} \tag{2.24}$$

and therefore

$$f(\delta E)^2 = f(2\delta E)\,, \tag{2.25}$$

which is not true unless $\alpha = -\infty$, and in particular is only valid for the Boltzman distribution (2.22). This does not indicate that no equilibrium will be achieved (as claimed in [21]). One can show that a system obeying (2.21) does equilibrate, but the configuration will not be consistent with detailed balance. Unitarity, time reversal invariance of the microphysics, and equipartition imply detailed balance; our result indicates that one of these breaks down when coupling a detector which possesses these properties to a field in the state $|\alpha\rangle$.

In the thermal vacuum, $\mathcal{R}_{ij}$ drops exponentially for $\delta E \gg 2\pi H$. But for *finite* α and $\delta E \gg \pi H$, the ratio asymptotes to

$$\mathcal{R} \to e^{2\alpha}\,.$$

For $\delta E \gg \pi H$, there is a constant probability for the detector to be excited, for all values of δE. This is consistent with $|\alpha\rangle$ having infinite energy. We will discuss this further in section 3 and the appendix.

2.1.4 The quantum stress-energy tensor

We are particularly interested in the case where ϕ is the inflaton. At distances shorter than $1/H$, it can be treated as a massless, minimally coupled scalar. Ignoring potential energy terms, the expectation value of the stress-energy tensor can be computed from the anticommutator:

$$T^{\alpha,\mathrm{unreg}}_{\mu\nu} = \lim_{x\to x'}\left(\partial_\mu\partial_\nu{}' - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\partial_\beta{}'\right)G^{(+)}_\alpha(x,x')\,, \tag{2.26}$$

where

$$G^{(+)}_\alpha = \langle\alpha|\left\{\phi(x),\phi(x')\right\}|\alpha\rangle\,. \tag{2.27}$$

Because of the short-distance singularity in $G^{(+)}$, (2.26) is divergent. In the limit $x \to x'$, the last two terms in (2.14) are irrelevant, so that the divergence in this quantity can be related to the divergence in the thermal vacuum [17, 18] :

$$T_{\mu\nu}^{\alpha,\mathrm{unreg}} = \frac{1 + \exp{(\alpha + \alpha^*)}}{1 - \exp{(\alpha + \alpha^*)}} T^{\mathrm{therm,unreg}} \tag{2.28}$$

In the thermal vacuum, sensible regularization procedures exist, and lead to a finite value for $T_{\mu\nu}$ (c.f. [34, 38]). We now turn to the question of whether a sensible procedure exists for finite α.

At proper distances shorter than the Hubble scale, the background approaches flat space, and the thermal vacuum approaches the Minkowski vacuum. The results of coupling an Unruh detector to $|\alpha \neq -\infty\rangle$ imply that such states are excitations above the Minkowski vacuum. At arbitrarily high energy these excitations do *not* fall off in amplitude, so that the quantum stress energy tensor will be infinite.

We may nonetheless try to regularize $T_{\mu\nu}$. One option, described in [4, 5] is to compute $T_{\mu\nu}^{\alpha}$ by expanding ϕ into creation and annihilation operators, and simply discarding the contributions of all modes with proper momenta higher than some cutoff scale Λ. The modes with lower momenta are taken to contribute to $T_{\mu\nu}$. As modes at the scale Λ inflate away, new modes carrying stress-energy must be "pumped into" the system at the scale Λ to maintain the deviations from the thermal vacuum. Prior to their being pumped in, they must not interact with gravity. This is unphysical.

A physical procedure for cutting off the theory would be to choose some particular time in inflating coordinates, and cut off the excitations above some momentum scale Λ. One must ask what "excitations" mean. The clearest definition, given our discussion of particle detectors, is excitations relative to the thermal vacuum. After all, the thermal vacuum approaches the standard Minkowski vacuum at short distances. In this case the additional excitations will inflate away in

$$N = \ln \frac{\Lambda}{H} \,. \tag{2.29}$$

Hubble times.

This regulator is not de Sitter invariant, however. One may instead choose a covariant, de Sitter-invariant regulator, such as a proper time cutoff or a covariant point-splitting regulator. (See [34, 38] for a detailed discussion). Such regulators subtract the short distance singularity in the definition of the composite operator $T_{\mu\nu}$. But we can see from (2.28) that the short distance behavior of the operator product defining $T_{\mu\nu}$ is different for different values of α. Similarly, the Unruh detector calculation leads to a state with a structure at high energies which depends on α.

Therefore, the prescription for cutting off the theory must depend on α if the vacuum energy is to remain finite. But each prescription for a short distance cutoff defines a different field theory. If we pick a theory for which $|\alpha\rangle$ is well defined, $|\beta \neq \alpha\rangle$ will not be well defined. When computing $T_\beta = \langle\beta|T_{\mu\nu}|\beta\rangle$, one would subtract the divergence of the OPEs in the vacuum $|\alpha\rangle$. From (2.28) it immediately follows that T_β is infinite.

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While one might fantasize about a theory consistent with a vacuum $|\alpha\rangle$ in eternal de Sitter space, in the standard inflationary scenarios inflation ends and the universe eventually relaxes to a state with the puzzlingly small cosmological constant that we observe. A theory which describes the world should be consistent with the Minkowski space vacuum at distances significantly shorter than the *present* Hubble scale. This theory must then be defined with a regulator which yields a zero-energy Minkowski vacuum in flat space. Therefore, either the state of the inflaton during inflation is a finite excitation above the thermal vacuum, and relaxes to it in a few e-foldings, or the short-distance structure of the theory must change with H during inflation. This latter scenario violates decoupling; the theory is nonlocal at the scale H. At this point we must simply throw up our hands. This kind of nonlocality does not appear in string theory, which is local down at least to the string scale M_s. So long as the string scale is higher energy than the Hubble scale, effective field theory is valid between these scales and decoupling will hold. If one allows $M_s \leq H$, as in [15], the de Sitter temperature is greater than or equal to the Hagedorn temperature. In this case the standard picture of inflation completely breaks down and the calculations in all works in question are invalid.[7]

2.2 The vacuum in the static patch

2.2.1 Coordinate system

The shaded region in figure 2 can be described by static coordinates, which are the analog of Schwarzchild coordinates in a black hole background. The metric is:

$$ds^2 = -\left(1 - \left(\frac{r}{R}\right)^2\right) dt_s^2 + \left(1 - \left(\frac{r}{R}\right)^2\right)^{-1} dr^2 + r^2 d\Omega_2^2 \,, \tag{2.30}$$

where $R = 1/H$.

A generalization of these coordinates also exists when the Hubble parameter changes with time [35], so that we can sensibly discuss the physics of inflation from the point of view of a single inertial observer. This is the analog in de Sitter space of studying physics from the point of view of an observer outside the black hole.

Eq. (2.30) describes a spherically symmetric geometry, with the origin surrounded by a horizon at $r = R$. The near-horizon limit of (2.30) is simply Rindler space; the horizon behaves much like the horizon of a Schwarzschild black hole. At $r = 0$, the time coordinate t_s is identical to the time coordinate in the inflating and global coordinates. Therefore, according to an inertial observer at $r = 0$, the vacuum has temperature T_{dS}.

For an observer at fixed $r \neq 0$, the local temperature blueshifts as r approaches R, and becomes infinite at the horizon. One may cut off the space by introducing a stretched horizon (rendering the entropy finite), as advocated in [26] for black holes. The stretched horizon can be defined, for example, as the surface a Planck length away from the true horizon. It can be thought of as a hot membrane with $T = M_p$.

[7]Given the mysteries of the cosmological constant and the vacuum energy, one might try to subtract off the varying vacuum energy by adjusting the inflaton potential. But the time at which the α state decays to the thermal vacuum is not tied to the vev of the inflaton, so it is very unclear how to implement this. This issue is related to the fine-tuning issues discussed elsewhere in this work.

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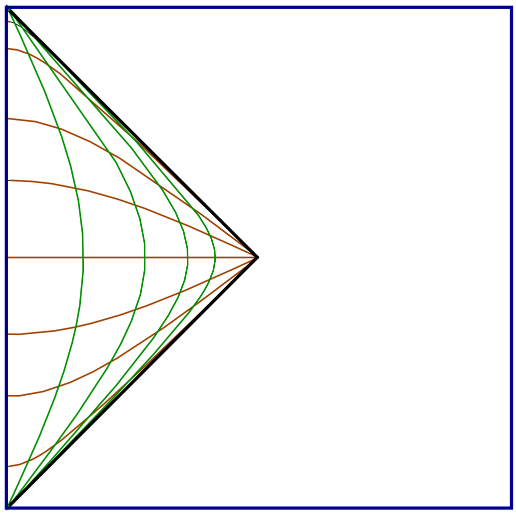

Figure 2: The static patch of de Sitter, with lines of constant time and radial coordinate depicted.

De Sitter space can therefore be described as a closed system with thermal walls, a "hot tin can." Such a system will always thermalize, in a time determined by the initial state. This is a very general statement, which does not depend on the details of the interactions or the spectrum of the hamiltonian. The only way it can be avoided is if the system is exactly free, so that it does not interact with the hot walls at all, or if it contains infinite energy densities, in which case the thermalization time can be taken to infinity.

Neither possibility is consistent with a theory coupled to gravity. In such theories, if an observer ever detects a finite-energy fluctuation originating from near the past horizon, it must have had a very large energy where it originated from, which was redshifted away during the propagation away from the horizon. Such a fluctuation will rapidly thermalize in the presence of gravity, which becomes very strong at high energies. In other words, take any finite energy fluctuation detected by our observer and boost it back to where it originated from. In a rest frame close to the horizon the fluctuation will be very high energy, therefore very short wavelength. Interactions with other such high energy quanta will very quickly produce a thermal distribution.

2.2.2 Thermalization time

Consider a state with some excitations in the vicinity of the origin, in the coordinates (2.30). If the excitation spectrum is thermal with $T = T_{dS}$, the excitations will be in equilibrium with the radiation from the stretched horizon.

The states $|\alpha\rangle$ are *not* thermal (see (2.21); t_s and t are identical at $r = \vec{x} = 0$.) In particular, the detector coupled to a field in the state $|\alpha\rangle$ is excited with finite and constant probability per unit time for arbitrarily large δE, due to the absorbtion of highly energetic particles. These energetic particles will generally quickly exit the region of the origin and arrive at the horizon. In the case at hand we can certainly describe them as relativistic. The inflaton will have mass $m \ll H$, while the excitations we describe have energy much greater than H, so we can ignore their rest mass.

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We can thus approximate the time for this excitation to travel from the origin to the stretched horizon, as the time along a massless geodesic:

$$t_s = \int_0^{R-L_p} \left(1 - \left(\frac{r}{R}\right)^2\right)^{-1} dr = \frac{R}{2} \ln \left(\frac{2R - L_p}{L_p}\right). \tag{2.31}$$

In other words, it will reach the horizon in $N = Ht = (1/2)\ln(2M_p/H) < 7$ e-foldings of inflation, using the most optimistic (for generating effects on the CMB) values of H and the cutoff. Note that this is roughly consistent with the estimate (2.29) for an excitation to inflate away in flat coordinates. (Massive particles will take longer to reach the horizon: they will accelerate exponentially away from the origin, until they reach a distance comparable to the Hubble length.)

When the particle arrives at the stretched horizon, it will be blueshifted to energies greater than M_p relative to an observer in the center. Since the temperature of the stretched horizon is M_p, gravitational interactions will be of order unity, and the excitation will immediately thermalize. Any non-gravitational interactions will generically only thermalize the particle more quickly.

If the state has an infinite number of arbitrarily high excitations, as is the case with $|\alpha\rangle$ *sans* regularization, this argument will fail. However, in this case backreaction will destroy the de Sitter geometry.

On the other hand, if the boundary conditions at the horizon are such that there is a non-zero flux of particles coming in or out, the naive argument could also fail. In this case, however, there will be constant collisions at the stretched horizon between the incoming and outgoing fluxes, or between the flux and the thermal background, with energies of order the Planck mass. This also will take the model out of perturbative control, as the back-reaction at the horizon will be large.

To avoid these problems, one could cut off the α state. This will generically break the de Sitter invariance, leaving a state of the type described above (so that the state will thermalize in ~ 10 e-foldings).

3. Inflation and $|\alpha\rangle$

The utility of inflation stems from its predictive power. The common lore is that "anything will inflate away," leaving a flat and homogeneous spacetime at the end of inflation, which matches our observations. But of course the universe is not perfectly homogeneous, and in fact the inclusion of quantum fluctuations in the theory provides a beautiful explanation for the CMBR anisotropy and the eventual formation of large-scale structure.

Observations *do* force a certain degree of fine-tuning in the effective field theory of the inflaton sector. In particular, the potential for the inflaton field must be quite flat, to allow the universe to expand superluminally (typically 60–70 e-foldings suffice), and to explain the small CMBR anisotropies $(\delta\rho)/\rho \sim 10^{-5}$. The predictivity of inflation then comes from the fact that, given this assumption, the initial conditions are completely irrelevant to the spectrum we observe today, because initial inhomogeneities are "inflated away".

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This solves the so-called horizon and flatness problems (but see [35] for a discussion of the genericity of inflationary initial conditions).

One can easily conjure models which satisfy the slow roll conditions and the usual assumptions of effective field theory, and yet which predict unusual features in the fluctuation spectrum, detectable by the next generation of precision CMBR experiments. For example, one may add a field which couples to the inflaton and which becomes light ($m \ll H$) between 70 and 50 e-foldings before the end of inflation [36, 6]. This will lead to a noticeable feature in the spectrum of CMBR anisotropies. A suitable choice of parameters takes this feature to the threshold of observability. One can even add a whole series of such fields, providing potentially infinite freedom to manipulate the predictions. Such models, while perhaps interesting in principle, are highly non-generic in that they require a degree of fine-tuning far above and beyond what is necessary merely to match the observed scale invariant spectrum. However, these models have the virtue that they are sensible effective field theories. Their shortcoming is of course that they are even more unnatural than the simplest inflationary models. In general, field theorists prefer not to give up naturalness unless they are forced to by the data.

Our discussion in the previous section indicates that the putative distortions in the CMBR spectrum which arise from considering the inflaton in a state $|\alpha \neq -\infty\rangle$ is as bad, or potentially worse, than the other fine tuning issues. Unregulated, the states are pathological. If we impose the simplest cutoffs, the states relax quickly to the thermal vacuum. We can get an observable effect in the CMBR only if we fine tune the initial conditions so that inflation begins precisely at 65 e-foldings, so that the distortions occur within the first ten e-foldings of inflation before they inflate away. But in general there is no reason that inflation should not last for 70 or 80 or 100 e-foldings. Furthermore, all conventional models of inflation assume the initial state is inhomogeneous to some degree. If inflation began precisely 65 e-foldings ago, the CMB spectrum would be distorted simply by the fact that the initial inhomogeneities in the inflaton would not have had time to be ironed out by the expansion.

We should note that while CMBR anisotropy measurements are sensitive to effects generated between 55-65 e-foldings before the end of inflation, gravitational wave detectors are sensitive to fluctuations generated much later. Therefore, even finely-tuned states will not affect these latter experiments.

Part of our argument that $|\alpha\rangle$ is sick rested on the observation that the universe today is nearly flat, and that the vacuum state is close to the Minkowski vacuum. In this section we will provide further arguments to show that the states $|\alpha\rangle$ are pathological in a realistic cosmology.

3.1 The causal structure of $|\alpha\rangle$

A worrysome aspect of the $|\alpha\rangle$ states is the presence of singularities in the Green functions at spacelike separated points (in particular, the Green functions diverge when the points are antipodal; see figure 3).

The retarded and advanced propagators do not suffer from this disease, as they depend only on the commutator of fields and as such are determined purely by the operator

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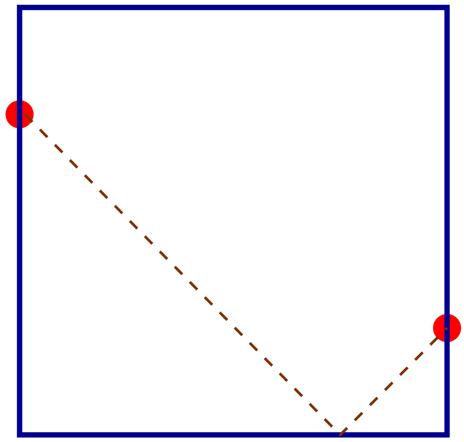

Figure 3: Two antipodal points in de Sitter.

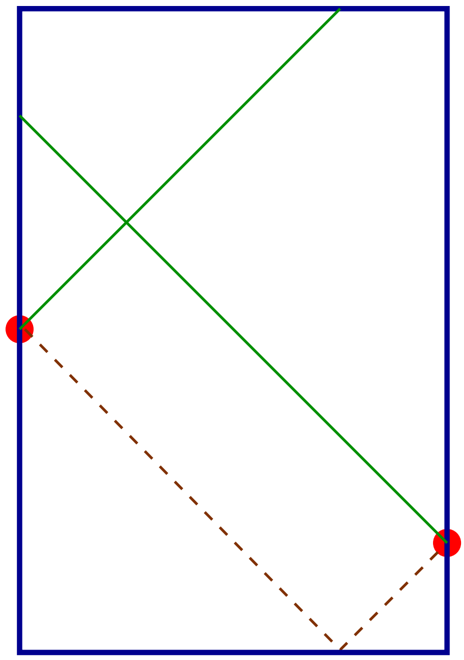

Figure 4: The Penrose diagram for a de Sitter space with some matter present, or for one which transitions from inflation (the bottom part of the diagram) to a final, larger de Sitter. In such a space the future lightcones of some of the antipodal points in the inflating patch can intersect.

algebra. For this reason we do not expect the theory to be acausal, despite the presence of spacelike singularities. Furthermore, in eternal de Sitter, the causal patch and its antipodal image only overlap at $t = \pm\infty$ (in other words, the antipodal point is always "behind the horizon"). However, any perturbation to exact de Sitter elongates the Penrose diagram, which brings some antipodal points into the past lightcone of the origin at finite time (see figure 4).

Although this apparently does not render the theory acausal, it is disturbing, and probably indicates in yet another way the pathological nature of the $|\alpha\rangle$ states.

Wald has argued [38, 37] that a vacuum in a QFT with states satisfying a standard positive norm condition, and the condition that the short distance behavior of the Green functions match flat space, is sufficient to guarantee the absence of such spacelike separated singularities. He therefore proposed as a criterion for curved space quantum field theory that the short distance behavior should match to flat space. The α states are examples of states which fail to meet this criteria.

3.2 Inflationary fluctuations, thermal vacuum and "other" Hubble volumes

When the thermal vacuum prescription is adopted, the standard causal dynamics for generation of inflationary fluctuations produces the seeds for large scale structure, and sources the CMB anisotropies. We have reviewed this classic result [40, 41, 48, 49, 50, 51] in our earlier work [6]. Here we underscore the crucial role of the thermal vacuum in particular for the generation of fluctuations.

The inflationary density fluctuations come from imprinting the quantum inflaton fluctuations on the background geometry. They are generated by the process of inflationary stretching of the fluctuations to the scales larger than the apparent horizon, where the fluctuations freeze, and remain decoupled until horizon reentry much after inflation. However the universe which we presently experience consists of a huge number of domains which were Hubble size at the time of inflation, and therefore causally disconnected. If we take the scale of inflation to be $H \sim 10^{14}$ GeV, then the ratio of the observable volume of the universe today to the volume then was

$$n_d \sim \frac{H_{\text{now}}^{-3}}{H_{\text{inflation}}^{-3}} \frac{a^3(\text{inflation})}{a^3(\text{now})} > 10^{78} \,. \tag{3.1}$$

While inflation evicted initial inhomogeneities from these domains, naively one may worry that the subsequent production of density fluctuations could reproduce inhomogeneities which are larger than those we observe, $\delta\rho/\rho \sim 10^{-5}$, because the inflaton fluctuated independently in each of the many Hubble regions.

Closer scrutiny reveals that this is not the case, and the thermal vacuum prescription plays a key role in this. Namely, the equivalence principle guarantees that the local physics in each Hubble domain is identical. However, the Hubble domains which are not centered around our observer are boosted relative to her, and thus in terms of her reference frame the phenomena in these domains occur at much higher proper energies, in exactly the same manner as we have discussed in section 2.1. Thus, in the thermal vacuum, the fluctuations originating in distant Hubble domains which may have finite energy in the center of the inflating patch will have a very small amplitude, because they had thermalized before reaching the observer. A vacuum state containing high energy excitations which are not exponentially suppressed, however, would not be protected by this mechanism and as such would be expected to produce large inhomogeneities. It is possible to arrange for some nonthermal effects to survive until the end of inflaton, if it is not eternal. However, as we have stressed above, this is just another guise of fine tuning, and therefore of little predictive value.

3.3 The α state in the post-inflationary universe

Let us assume for a moment that the in-principle difficulties the state $|\alpha\rangle$ can be resolved with a suitable de Sitter invariant regulator, so that the state does not relax to the thermal vacuum.

In section 2.1 we reminded the reader that correlation functions in the state $|\alpha\rangle$ have short-distance behavior which differs from the behavior in the thermal vacuum or in flat space. In order to make the stress tensor finite in the state $|\alpha\rangle$, we must cut off the theory in a *different* way than we would in flat space. However, the epoch of inflation must have come to an end; the universe today has a tiny cosmological constant and nearly zero curvature. This causes a serious problem with basic (e.g. atomic) physics.

The authors of [4, 5, 13, 14, 16] considered as a state for the inflaton $|\alpha\rangle$, with, for example, $e^\alpha = H_i/M \sim 10^{-5}$ where H_i is the Hubble constant during inflation. Imagine a hypothetical Unruh detector in this background. As we demonstrate in the appendix, the population densities at equilibrium are not thermal, and in fact tend to a constant at high energies. The deviation from thermality becomes significant for energies E satisfying $E \gg H\alpha$. During inflation, $H_i\alpha = H_i \ln(M/H_i)$, which is large, and hence if any sensible regulation of the α vacuum were possible, the distortion from the thermal spectrum below this scale will not be serious. Indeed, it is precisely this small distortion which is argued to lead to interesting (order H/M) corrections to the CMB spectrum, in conflict with the effective field theory analysis of [6].

However, let us consider the universe today, with H_t being the Hubble constant today. The question is, what is the state of the inflaton today? Unless we assume that the short-distance structure is non-local on a scale set by the Hubble constant of the universe, so that α is H dependent, it will still be in the same α state today that it was during inflation.[8]

In this case basic atomic physics is impossible! The condition

$$\frac{E}{H_t} \gg \alpha \tag{3.2}$$

implies that

$$E \gg \alpha \times 10^{-31}\,\mathrm{eV}\,, \tag{3.3}$$

(recall that $\exp(\alpha) \sim 10^{-5}$ to produce interesting effects on the CMB spectrum). Consider, for example, a transition from the ground state to any excited state with an energy greater than 10^{-31} eV. The rate will be approximately $e^{2\alpha} \sim 10^{-10}$, which is *independent* of the energy of the excited state. Far worse, however, the population density of states in this equilibrium will be roughly constant, at least up to an energy of order $e^{2\alpha}M \sim 10^9$ GeV. This is obviously a disaster. In particular, a hydrogen atom coupled to fields in this vacuum will be ionized with probability equal to unity.

[8] Of course, it has been proposed by many authors that a solution to the cosmological constant problem requires mixing between the scale of the present horizon and the fundamental UV scales of the theory. However, the best guess for the effects of this mixing during inflation would be that they make the number of degrees of freedom in an inflating patch finite [42]–[44]. The effects on observations will be much smaller than the effects discussed here [45].

To avoid this, one should require $e^{\alpha} = H_t/M_p$. However, this gives effects on the CMB spectrum of one part in 10^{60}, which are obviously totally unobservable. The only other option is to require that α change with time as the Hubble constant changes, for example so as to always satisfy $e^{\alpha(t)} = H(t)/M_p$. As we have argued, this is a massive violation of decoupling, and implies that the theory is nonlocal on the scale $H(t)$.

4. Conclusions

In this note we have reviewed the choice of ground state in de Sitter space and in an inflating universe with a slowly rolling inflaton. It has been suggested that de Sitter symmetries allow one to pick any de Sitter-invariant state $|\alpha\rangle$. We have argued, however, that any $\alpha \neq -\infty$ is only consistent if the field theory is completely free, and backreaction is ignored. All other excitations inflate away or thermalize rapidly. This is similar to the situation familiar in the black hole physics, where the thermal vacuum is the choice consistent both with effective field theory and holography.

From the standpoint of inflationary cosmology, this choice then implies that for computations of density perturbations generated during inflation, the correct procedure is to use local effective field theory in the thermal vacuum, as in [6]. The high-energy corrections to inflationary density fluctuations will come in the form of an expansion in even powers of H/M, as has been discussed in [6]. We *are* assuming that the scale at which the theory becomes nonlocal is shorter than the Hubble scale, but if not, the scenario will be rather different from standard inflation and no calculations performed to date will be applicable.

We close with a few additional comments.

4.1 Higher-dimension operators and inflationary perturbations

Within the context of effective field theory, Shiu and Wasserman [39] have recently pointed out a class of higher-derivative terms which lead to potentially larger corrections than those described in [6].

In general, terms of the form

$$\delta_{2n}\mathcal{L} = \frac{1}{\Lambda^{4n-4}} (\partial\phi)^{2n} \ , \tag{4.1}$$

will appear in the inflaton effective action, with Λ the scale of the physics which controls such operators. The even powers are required by Lorentz invariance. Then during inflation, substituting $\phi = \phi_0(t) + \tilde{\phi}$ into (4.1), where $\phi_0(t)$ is the homogenous mode of the inflaton, using the slow roll conditions,

$$\frac{(\dot{\phi})^2}{\Lambda^4} = 2\epsilon \left(\frac{m_4^2}{\Lambda^2}\right)\left(\frac{H^2}{\Lambda^2}\right), \tag{4.2}$$

we find htat $\delta_4\mathcal{L}$ leads to a wavefunction renormalization of the form:

$$\delta\mathcal{L} = \left(\frac{m_4^2}{\Lambda^2}\right)\left(\frac{H^2}{\Lambda^2}\right)\left(\partial\tilde{\phi}\right)^2 . \tag{4.3}$$

This is consistent with [6], in the sense that it is an even power of H/Λ. The point of [39] is that the *coefficient* is proportional to m_4^2/Λ^2, which may be large. Along the lines of the arguments in [6], this would yield observably large corrections to the consistency condition for n_T if $\Lambda \sim M_{\rm GUT}$. For example, the Kaluza-Klein modes in Hořava-Witten models consistent with grand unification would generically have observable effects on the spectrum of CMBR anisotropies. This is an exciting prospect.

Unfortunately, we believe that in realistic theories one should not equate Λ in (4.1) to the fundamental scale of the theory. For example, in M-theory compactifications, the inflaton ϕ is naturally a compactification modulus. Following the line of reasoning in [46, 47], we find that $\Lambda \sim \sqrt{m_{11} m_4}$, as follows.

The inflaton naturally arises as a dimensionless scalar ψ because it is a component of the eleven-dimensional metric. In the four-dimensional effective action, the kinetic term and the terms (4.1) will take the form:

$$\mathcal{L}_{kin} = m_4^2\, (\partial\psi)^2\, \mathcal{F}\left(\frac{(\partial\psi)^2}{m_{11}^2}\right), \tag{4.4}$$

where $\mathcal{F}(0) = 1$. Here m_{11} is the eleven-dimensional Planck scale. The factor of m_4^2 in front arises from the large volume of the compactification, in the same way that it appears in front of the four-dimensional Einstein action. Defining $\phi = m_4 \psi$, to normalize ϕ canonically in the effective 4d theory, (4.4) becomes

$$\mathcal{L}_{kin} = (\partial\phi)^2\, \mathcal{F}\left(\frac{(\partial\phi)^2}{m_4^2 m_{11}^2}\right). \tag{4.5}$$

As we claimed, $\Lambda \sim \sqrt{m_{11} m_4}$. We intend to investigate this further in various contexts such as brane-world scenarios and warped compactifications.

Acknowledgments

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A. Thermodynamics of the α states

As we discussed in section 2, the probability per unit time $\dot{P}_{\alpha,i\to j}$ for an excitation of an Unruh detector, coupled to a field in an α state, to transition from a state E_i to a state E_j, can be expressed

$$\dot{P}_{\alpha,i\to j} = |\langle E_j|m(0)|E_i\rangle|^2 \int_{-\infty}^{\infty} dt e^{-i\delta E t} G_\alpha(t) \equiv |\langle E_j|m(0)|E_i\rangle|^2 \mathcal{F}_\alpha(\Delta E)\,. \tag{A.1}$$

Here $\mathcal{F}_\alpha(E)$ is the "response function" of the detector. It depends only on the state of the scalar, not on any internal details of the detector, and hence is a useful diagnostic for the nature of the state. Using general properties of the Green function under imaginary time shifts, one obtains (at least in 2+1 [21] and 3+1 dimensions) the ratio

$$\mathcal{R}_{ij} = \frac{\dot{P}_{i\to j}}{\dot{P}_{j\to i}} = e^{-2\pi\Delta E/H}\left|\frac{1+e^{\alpha+\pi\Delta E/H}}{1+e^{\alpha-\pi\Delta E/H}}\right|^2 . \tag{A.2}$$

In a system which equilibrates in accord with the principle of detailed balance, this ratio suffices to determine the relative population densities. However, as we argued in section 2, the equilibrium reached by a system in contact with an α state will not be consistent with detailed balance. In this appendix, we will compute $\mathcal{F}_\alpha$ exactly for the case of a massless, conformally coupled scalar in 3+1 de Sitter (which is identical for these purposes to a minimally coupled scalar with mass $\sqrt{2}H$).

For such a scalar, the positive frequency Green function in the thermal vacuum is

$$G_0(x,x') = \frac{-H^2}{4\pi^2}\frac{\eta\eta'}{[(\Delta\eta)^2 - |\Delta\mathbf{x}|^2]}, \tag{A.3}$$

where $\eta = -e^{-Ht}/H$. Using the identity $\pi^2\sin^{-2}(\pi x) = \sum_{n=-\infty}^{\infty}(x-n)^{-2}$, and setting $\Delta\mathbf{x} = 0$,

$$\begin{aligned}\mathcal{F}_0(E) &= \frac{H^2}{32\pi^2}\int_{-\infty}^{\infty} d(t+t')\int_{-\infty}^{\infty} d\Delta t \frac{e^{-iE\Delta t}}{\sin^2(iH\Delta t/2+\epsilon)} \\ &= \frac{H^2}{32\pi^4}\int_{-\infty}^{\infty} d(t+t')\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta t\frac{e^{-iE\Delta t}}{(iH\Delta t/2\pi+\epsilon-n)^2},\end{aligned} \tag{A.4}$$

where we have included an ϵ prescription to avoid the pole at $\Delta t = 0$. This integral can now be computed either by completing the contour above or below the real axis. The residues of the integrand are $-(4\pi^2 iE/H^2)e^{-2\pi nE/H}$. If $\epsilon > 0$ (which is the correct prescription) and we complete the contour below the real axis, we miss the pole near the origin, and obtain a sum of the form

$$\dot{\mathcal{F}}_0(E) = \left(\frac{E}{2\pi}\right)\sum_{n=1}^{\infty} e^{-2\pi nE/H} = \left(\frac{E}{2\pi}\right)(e^{2\pi E/H}-1)^{-1} . \tag{A.5}$$

This is a Planck distribution, which is consistent with the thermal character of the standard de Sitter vacuum. On the other hand, had we chosen the incorrect ϵ prescription, we would have picked up the pole at the origin, obtaining

$$\dot{\mathcal{F}}_-(E) = \sum_{n=0}^{\infty} e^{-2\pi nE/H} = \left(\frac{E}{2\pi}\right)\left(1-e^{-2\pi E/H}\right)^{-1} . \tag{A.6}$$

This is an unphysical distribution, which blows up at large energies.

In an α state, the Green function can be expressed as (2.14)

$$G_\alpha(x,x') = |N_\alpha|^2 \Bigg[G_0(x,x') + \exp(\alpha+\alpha^*)\, G_0(x',x) + \\ + \exp(\alpha^*) G_0(x,x'_A) + \exp(\alpha) G_0(x_A,x') \Bigg] . \qquad (A.7)$$

The second term involves the wrong ordering of x, x', so that $\Delta t \to -\Delta t$, which is equivalent to changing the sign of ϵ, and therefore contributes a term proportional to (A.6). The terms involving the antipodal points can be evaluated using the identity $G_0(x,x'_A) = G_0(x_A,x') = G_0(t - i\pi/H)$, for x, x' at the origin. From these we obtain

$$\int_{-\infty}^{\infty} d\Delta t \frac{e^{-iE\Delta t}}{\sin^2\left(iH\Delta(t-i\pi/H)/2\right)} = \frac{1}{\pi^2}\sum_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta t \frac{e^{-iE\Delta t}}{(iH\Delta t/(2\pi)+1/2-n)^2} \\ = (2E/\pi) e^{\pi E/H}(e^{2\pi E/H}-1)^{-1} . \qquad (A.8)$$

Plugging everything back into (A.7), we obtain the response function for arbitrary α:

$$\begin{aligned} \dot{\mathcal{F}}_\alpha(E) &= N_\alpha^2 \frac{(E/2\pi)}{e^{2\pi E/H}-1}\left[1 + e^{\alpha+\alpha^*+2\pi E/H} + e^{\alpha+\pi E/H} + e^{\alpha^*+\pi E/H}\right] \\ &= N_\alpha^2 \left|1 + e^{\alpha+\pi E/H}\right|^2 \frac{(E/2\pi)}{e^{2\pi E/H}-1} \\ &= N_\alpha^2 \left|1 + e^{\alpha+\pi E/H}\right|^2 \mathcal{F}_0(E) , \end{aligned} \qquad (A.9)$$

in exact agreement with (A.2), and with the result obtained in [21] for the case of 2+1 dimensional de Sitter.

As we discussed in section 2, such a distribution is not consistent with detailed balance, and the density of states of a detector coupled to it appears to depend on the details of the detector (contrary to the thermal case). However, we can make a general statement (assuming the matrix elements $\langle E_j|m(0)|E_i\rangle$ are not zero; that is, that the detector is capable of being excited to these energies) about the equilibrium that a system coupled to a scalar in the α state will reach. The general condition for equilibrium is

$$\int_{E_0} \left(\rho(E')\dot{P}_{E'\to E} - \rho(E)\dot{P}_{E\to E'}\right) dE' = 0 . \qquad (A.10)$$

For an ordinary Planck distribution, $\dot{P}$ dies exponentially at large, positive energies. The α distribution (A.9), however, grows linearly with energy, both for large positive and negative ΔE. Therefore, the high energy density of states $\rho(E')$ in the first term in (A.10) must tend to a constant at large energies, in order to cancel the second term, which will pick up an infinite contribution in the UV. In other words, the high-energy population densities tend to a constant, rather than falling off exponentially with the energy as would be the case in a thermal distribution.

References

[1] J. Martin and R.H. Brandenberger, *The trans-planckian problem of inflationary cosmology*, *Phys. Rev.* **D 63** (2001) 123501 [hep-th/0005209];
R.H. Brandenberger and J. Martin, *The robustness of inflation to changes in super-Planck-scale physics*, *Mod. Phys. Lett.* **A 16** (2001) 999 [astro-ph/0005432];
R.H. Brandenberger, S.E. Joras and J. Martin, *Trans-planckian physics and the spectrum of fluctuations in a bouncing universe*, *Phys. Rev.* **D 66** (2002) 083514 [hep-th/0112122].

[2] J.C. Niemeyer, *Inflation with a high frequency cutoff*, *Phys. Rev.* **D 63** (2001) 123502 [astro-ph/0005533];
J.C. Niemeyer and R. Parentani, *Trans-planckian dispersion and scale-invariance of inflationary perturbations*, *Phys. Rev.* **D 64** (2001) 101301 [astro-ph/0101451].

[3] A. Kempf and J.C. Niemeyer, *Perturbation spectrum in inflation with cutoff*, *Phys. Rev.* **D 64** (2001) 103501 [astro-ph/0103225].

[4] R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, *Inflation as a probe of short distance physics*, *Phys. Rev.* **D 64** (2001) 103502 [hep-th/01044102].

[5] R. Easther, B.R. Greene, W.H. Kinney and G. Shiu, *Imprints of short distance physics on inflationary cosmology*, hep-th/0110226.

[6] N. Kaloper, M. Kleban, A.E. Lawrence and S. Shenker, *Signatures of short distance physics in the cosmic microwave background*, hep-th/0201158.

[7] L. Hui and W.H. Kinney, *Short distance physics and the consistency relation for scalar and tensor fluctuations in the inflationary universe*, *Phys. Rev.* **D 65** (2002) 103507 [astro-ph/0109107];
For examples of other models that discuss modifications of the consistency conditions, see e.g., J. Garriga and V.F. Mukhanov, *Perturbations in K-inflation*, *Phys. Lett.* **B 458** (1999) 219 [hep-th/9904176];
G. Shiu and S.H.H. Tye, *Some aspects of brane inflation*, *Phys. Lett.* **B 516** (2001) 421 [hep-th/0106274].

[8] D. Polarski and A.A. Starobinsky, *Structure of primordial gravitational waves spectrum in a double inflationary model*, *Phys. Lett.* **B 356** (1995) 196 [astro-ph/9505125];
J. Garcia-Bellido and D. Wands, *Constraints from inflation on scalar-tensor gravity theories*, *Phys. Rev.* **D 52** (1995) 6739 [gr-qc/9506050];
M. Sasaki and E.D. Stewart, *A general analytic formula for the spectral index of the density perturbations produced during inflation*, *Prog. Theor. Phys.* **95** (1996) 71 [astro-ph/9507001].

[9] N. Bartolo, S. Matarrese and A. Riotto, *Adiabatic and isocurvature perturbations from inflation: power spectra and consistency relations*, *Phys. Rev.* **D 64** (2001) 123504 [astro-ph/0107502].

[10] A.D. Linde, *Generation of isothermal density perturbations in the inflationary universe*, *Phys. Lett.* **B 158** (1985) 375;
L.A. Kofman, *What initial perturbations may be generated in inflationary cosmological models*, *Phys. Lett.* **B 173** (1986) 400;
L.A. Kofman and A.D. Linde, *Generation of density perturbations in the inflationary cosmology*, *Nucl. Phys.* **B 282** (1987) 555;

A. Linde and V. Mukhanov, *Non-gaussian isocurvature perturbations from inflation*, *Phys. Rev.* **D 56** (1997) 535 [astro-ph/9610219];
P.J.E. Peebles, *An isocurvature cdm cosmogony, 1. A worked example of evolution through inflation*, astro-ph/9805194.

[11] G. Efstathiou and J.R. Bond, *Isocurvature cold dark matter fluctuations*, *Mon. Not. Roy. Astron. Soc.* **218** (1986) 103.

[12] D. Langlois, *Correlated adiabatic and isocurvature perturbations from double inflation*, *Phys. Rev.* **D 59** (1999) 123512;
D. Langlois and A. Riazuelo, *Correlated mixtures of adiabatic and isocurvature cosmological perturbations*, *Phys. Rev.* **D 62** (2000) 043504 [astro-ph/9912497];
C. Gordon, D. Wands, B.A. Bassett and R. Maartens, *Adiabatic and entropy perturbations from inflation*, *Phys. Rev.* **D 63** (2001) 023506 [astro-ph/0009131];
N. Bartolo, S. Matarrese and A. Riotto, *Oscillations during inflation and cosmological density perturbations*, *Phys. Rev.* **D 64** (2001) 083514 [astro-ph/0106022].

[13] U.H. Danielsson, *A note on inflation and transplanckian physics*, *Phys. Rev.* **D 66** (2002) 023511 [hep-th/0203198].

[14] R. Easther, B.R. Greene, W.H. Kinney and G. Shiu, *A generic estimate of trans-planckian modifications to the primordial power spectrum in inflation*, *Phys. Rev.* **D 66** (2002) 023518 [hep-th/0204129].

[15] R.H. Brandenberger and J. Martin, *On signatures of short distance physics in the cosmic microwave background*, *Int. J. Mod. Phys.* **A 17** (2002) 3663 [hep-th/0202142].

[16] U.H. Danielsson, *Inflation, holography and the choice of vacuum in de Sitter space*, *J. High Energy Phys.* **07** (2002) 040 [hep-th/0205227].

[17] E. Mottola, *Particle creation in de Sitter space*, *Phys. Rev.* **D 31** (1985) 754.

[18] B. Allen, *Vacuum states in de Sitter space*, *Phys. Rev.* **D 32** (1985) 3136.

[19] N.A. Chernikov and E.A. Tagirov, *Quantum theory of scalar fields in de Sitter spacetime*, *Ann. Inst. Henri Poincaré* **A9** (1968) 109;
E.A. Tagirov, *Consequences of field quantization in de Sitter type cosmological models*, *Ann. Phys. (NY)* **76** (1973) 561.

[20] J. Géhéniau and C. Schomblond, *Acad. R. Belg. Bull. Cl. Sci.* **54** (1968) 1147;
C. Schomblond and P. Spindel, *Unicity conditions of the scalar field propagator $\Delta_1(X,Y)$ in de Sitter universe. (In french)*, *Ann. Inst. Henri Poincaré* **A25** (1976) 67.

[21] R. Bousso, A. Maloney and A. Strominger, *Conformal vacua and entropy in de Sitter space*, *Phys. Rev.* **D 65** (2002) 104039 [hep-th/0112218].

[22] N.J. Cornish, D.N. Spergel and C.L. Bennett, *Journey to the edge of time: the great mission*, astro-ph/0202001.

[23] N. Seta, S. Kawamura and T. Nakamura, *Possibility of direct measurement of the acceleration of the universe using 0.1 Hz band laser interferometer gravitational wave antenna in space*, *Phys. Rev. Lett.* **87** (2001) 221103 [astro-ph/0108011].

[24] C. Ungarelli and A. Vecchio, *High energy physics and the very-early universe with lisa*, *Phys. Rev.* **D 63** (2001) 064030 [gr-qc/0003021].

[25] W.G. Unruh, *Notes on black hole evaporation*, *Phys. Rev.* **D 14** (1976) 870.

JHEP11(2002)037

[26] L. Susskind, L. Thorlacius and J. Uglum, *The stretched horizon and black hole complementarity*, *Phys. Rev.* **D 48** (1993) 3743 [hep-th/9306069].

[27] T. Banks and L. Mannelli, *de Sitter vacua, renormalization and locality*, hep-th/0209113.

[28] M.B. Einhorn and F. Larsen, *Interacting quantum field theory in de Sitter vacua*, hep-th/0209159.

[29] K. Goldstein and D.A. Lowe, *Initial state effects on the cosmic microwave background and trans-planckian physics*, hep-th/0208167.

[30] E.W. Kolb and M.S. Turner, *The early universe*, Addison-Wesley, 1990.

[31] A. Linde, *Particle physics and inflationary cosmology*, Harwood Academic, 1990.

[32] A. Liddle and D. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press Cambridge 2000.

[33] For a review see J.E. Lidsey et al., *Reconstructing the inflaton potential — an overview*, *Rev. Mod. Phys.* **69** (1997) 373.

[34] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press Cambridge 1982.

[35] L. Dyson, M. Kleban and L. Susskind, *Disturbing implications of a cosmological constant*, *J. High Energy Phys.* **10** (2002) 011 [hep-th/0208013].

[36] D.J.H. Chung, E.W. Kolb, A. Riotto and I.I. Tkachev, *Probing planckian physics: resonant production of particles during inflation and features in the primordial power spectrum*, *Phys. Rev.* **D 62** (2000) 043508 [hep-ph/9910437].

[37] M.J. Radzikowski, *Micro-local approach to the hadamard condition in quantum field theory on curved space-time*, *Commun. Math. Phys.* **179** (1996) 529.

[38] R.M. Wald, *Quantum field theory in curved spacetime and black hole thermodynamics*, University Chicago Press, Chicago 1994.

[39] G. Shiu and I. Wasserman, *On the signature of short distance scale in the cosmic microwave background*, *Phys. Lett.* **B 536** (2002) 1 [hep-th/0203113].

[40] V. Mukhanov and G. Chibisov, *Quantum fluctuations and a nonsingular universe*, *Sov. Phys. JETP Lett.* **33** (1981) 532.

[41] A.D. Linde, *Scalar field fluctuations in expanding universe and the new inflationary universe scenario*, *Phys. Lett.* **B 116** (1982) 335.

[42] C.J. Hogan, *Holographic discreteness of inflationary perturbations*, *Phys. Rev.* **D 66** (2002) 023521 [astro-ph/0201020]; *Observing quanta on a cosmic scale*, *Fortschr. Phys.* **50** (2002) 694 [astro-ph/0201021].

[43] T. Banks, *Cosmological breaking of supersymmetry or little lambda goes back to the future, 2*, hep-th/0007146.

[44] W. Fischler, *Taking de Sitter seriously*, talk given at *Role of scaling laws in physics and biology (Celebrating the 60th birthday of Geoffrey West)*, Santa Fe, Dec. 2000.

[45] M. Kleban, S.H. Shenker and S. Thomas, unpublished.

[46] T. Banks, *Remarks on M-theoretic cosmology*, hep-th/9906126.

JHEP11(2002)037

[47] T. Banks, *M-theory and cosmology*, hep-th/9911067.

[48] A.A. Starobinsky, *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*, *Phys. Lett.* **B 117** (1982) 175.

[49] S.W. Hawking, *The development of irregularities in a single bubble inflationary universe*, *Phys. Lett.* **B 115** (1982) 295.

[50] A.H. Guth and S.Y. Pi, *Fluctuations in the new inflationary universe*, *Phys. Rev. Lett.* **49** (1982) 1110.

[51] J.M. Bardeen, P.J. Steinhardt and M.S. Turner, *Spontaneous creation of almost scale-free density perturbations in an inflationary universe*, *Phys. Rev.* **D 28** (1983) 679.